Acoustic Emission Source Location Using Multi-Frequency Arrival Times

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Abstract—In this paper, a method for a direct estimation of the source-sensor distances and the time of an Acoustic Emission event is presented using multiple sensors and the arrival-times for a certain number of frequencies. The mean square error (MSE) problem is solved directly for any set of available data. It is also proved that in case of error-free arrival-times, the source position can be derived analytically as a function of the material constant.

I. INTRODUCTION

Acoustic emission (AE) methods are based on the processing and information extraction from the ultrasonic waves, generated by pressure forces and damage events and it is used for monitoring, damage location and fracture analysis of composites using array of sensors [1-4]. A great number of studies are presented and a number of commercial systems are operating in real constructions for real-time identification and AE source location. Based on the pair of differences in arrival time, Tobias [5] derives the exact solution for three-sensor configuration in a planar surface at the intersection points of two hyperbolae. More precise source location has been achieved within the area bounded by the sensors. In some sensor configurations, ambiguous source positions can be appeared, i.e. the same difference in arrival time of emission waves at three sensors can arise from two distinct source locations. The source-sensor distance can also be estimated using the cross-correlation function of the AE signal with a single frequency cosine wave modulated by a Gaussian pulse [3], or by predicting the dispersion characteristics of the AE wave modes and involving wavelets [9,16].

In [6], the source position is calculated through the use of special coordinates on the surface of the pressure vessel and planar surfaces by increasing the radius of the sphere to infinity from three sensors [6]. In both types of surfaces, the location of the sources may be ambiguous and a fourth sensor is needed. In a similar approach Barat et al [8], developed a mathematical method for the calculation of the coordinates of an AE source on cylindrical surfaces on the concept of geodesics. In [7], a generalized mathematical formalism for the path traveled by an acoustic wave in the surface of a structure, on the concept of geodesics and a direct method for the calculation of the coordinates of an AE source on a cylindrical surface by three AE sensors is presented.

The source location in practice is an over-determined problem and its solution is defined statistically, using the popular least squares and the absolute value methods. In an extensive study [10] is shown that the location method, based on the absolute error often provides a more accurate result than the least squares method. In [11], the location of the friction source was successfully detected based on an accurate estimation of the arrival-time and derived from the phase shift of the cross-correlation function of two-sensor signals. The experimental results show, that the system is effective to detect friction during the deep drawing of sheet metal. In [12], a method which utilizes the entire AE waveform (viewed in the time–frequency domain) determine both the time of signal activation as well as the source–detector separation. In [13], a new algorithm allows the calculation of the 50% ellipse and two alternative, but equivalent procedures for the best estimation of the source location, corresponding to the ellipse’s centre for three or more microphone clusters. The maxima of the wavelet magnitude are used to calculate the arrival times of the waves and the coordinates of the impact location in [14]. The time lag and the group velocity of the flexural waves are obtained, by solving a system of four non-linear equations.

By taking into account the modal nature of AE signals, the number of sensors needed in AE source location can be reduced, as shown in [15]. Using signals obtained during tensile and bending tests performed on a number of cross-ply and unidirectional carbon fibre reinforced polymer (CFRP) lay-ups, it will be shown how a linear source location can be calculated using one sensor. To achieve this goal, two different plate wave theories will be used, and the results are compared to the ones obtained by a traditional two sensors linear location scheme.
In this work, the estimation of the source-sensors distance and the AE event-time is solved directly, as a square-error minimization problem, using multiple sensors and estimations of the arrival times at different frequencies. Moreover, the source location problem for three sensors can be derived at the minimum of a MSE function, and the corresponding gradient iterative methods can be extremely efficient. It is also proved that in case of error-free estimation of the real arrival-times, the source position can be derived analytically as a function of the material constant.

II. SOURCE-SENSORS DISTANCE ESTIMATION

If the group velocity of a frequency \( \omega \) is \( c(\omega) = Gf(\omega) \) as in [3], where \( G \) is constant depended on the material’s properties, the frequency arrival time at the sensor with distance \( r \) from the source is:

\[
t(\omega,r) = \frac{r}{Gf(\omega)} = \frac{r}{G} q(\omega) = Rq(\omega),
\]

where \( R \) is the normalized distance between the source and the sensor. If any of \( r, G, \) or even both of them, can be considered as unknown, a new variable \( R \) can replace the ratio \( r/G \).

A. Event time and source-sensor distance

If the group velocity \( c(\omega_n), n=1,N \) at \( N \) frequencies \( \omega_n \) is known, the accumulated error between the model-based arrival-times at the \( N \) frequencies of the AE event \( t(R,\omega_n) = Rq(\omega_n) \) and the real arrival-time of the signal at the same frequencies for an AE source, positioning in distance \( r \) from sensor, is:

\[
E(R,T) = \sum_{n=1}^{N} (t(R,\omega_n) + T - O_n)^2,
\]

where \( T \) is the time-difference between the event and the data acquisition starting time, and \( O_n \) is the real time-of-arrival of the \( \omega_n \) frequency component at the sensor.

In order to minimize the error the derivatives with respect to \( T \) and \( R \) must be zero:

\[
\frac{\partial E}{\partial R} = 0 \Rightarrow \sum_{n=1}^{N} (Rq_n + T)q_n = \sum_{n=1}^{N} O_n q_n,
\]

\[
\frac{\partial E}{\partial T} = 0 \Rightarrow \sum_{n=1}^{N} (Rq_n + T) = \sum_{n=1}^{N} O_n.
\]

In matrix form, the linear system of equations becomes:

\[
\begin{bmatrix}
\sum_{n=1}^{N} q_n^2 & \sum_{n=1}^{N} q_n \\
\sum_{n=1}^{N} q_n & N
\end{bmatrix}
\begin{bmatrix}
R \\
T
\end{bmatrix}
=
\begin{bmatrix}
\sum_{n=1}^{N} O_n q_n \\
\sum_{n=1}^{N} O_n
\end{bmatrix},
\]

or

\[
\begin{bmatrix}
S_q^2 & S_q \\
S_q & N
\end{bmatrix}
\begin{bmatrix}
R \\
T
\end{bmatrix}
=
\begin{bmatrix}
O^T Q \\
S_0
\end{bmatrix},
\]

where, \( O^T = [O_1 \ldots O_N] \), \( S_q = \sum_{n=1}^{N} q_n \), \( S_0 = \sum_{n=1}^{N} O_n \), \( S_q^2 = \sum_{n=1}^{N} q_n^2 \) and \( Q^T = [q(\omega_1) \ldots q(\omega_N)] = [q_n] \).

The source-sensor distance \( (R) \), and the event time \( (T) \) are estimated directly as follows:

\[
R = \frac{N \sum_{n=1}^{N} q_n - S_q S_0}{N S_q^2 - S_q^2}, \quad T = \frac{S_q^2 S_0 - O^T Q S_q}{N S_q^2 - S_q^2}.
\]

B. Event time and source-sensor distances using an array of sensors

In case of \( M \) sensors, the square error between the model-based arrival-times and the real arrival-times becomes:

\[
E(R_1,\ldots,R_M,T) = \sum_{m=1}^{M} \sum_{n=1}^{N} (t(R_m,\omega_n) + T - O_{mn})^2.
\]

The direct solution to the quadratic minimization problem is:

\[
\frac{\partial E}{\partial R_p} = 0 \Rightarrow \sum_{n=1}^{N} (R_p q_n + T)q_n = \sum_{n=1}^{N} O_{pm} q_n,
\]

\[
\frac{\partial E}{\partial T} = 0 \Rightarrow \sum_{n=1}^{M} \sum_{m=1}^{N} (R_mq_n + T) = \sum_{n=1}^{M} \sum_{m=1}^{N} O_{mn}.
\]

In matrix form:
The solution of the linear system of equations described by (11), can be re-written as:

\[
\begin{bmatrix}
    S_q & 0 & \ldots & 0 & S_q \\
    0 & S_q & \ldots & 0 & S_q \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    0 & 0 & \ldots & S_q & S_q \\
    S_q & S_q & \ldots & S_q & NM
\end{bmatrix}
\begin{bmatrix}
    R_i \\
    R_i \\
    \vdots \\
    R_i \\
    S_o
\end{bmatrix}
= \begin{bmatrix}
    O_p^T \mathbf{Q} \\
    O_p^T \mathbf{Q} \\
    \vdots \\
    O_p^T \mathbf{Q} \\
    S_o
\end{bmatrix}
\]

where \( O_p = [ O_{p1}, O_{p2}, \ldots, O_{pN} ] \), and \( S_o = \sum_{n=1}^{M} O_{mn} \).

The solution of the linear system of equations becomes:

\[
R_p = \frac{S_p^2 \sum_{n=1}^{M} O_{pq} + (N S_q^2 - M S_q^2) O_p^T \mathbf{Q} - S_q S_s S_o}{M S_q^2 (N S_q^2 - S_q^2)}
\]

\[
T = \frac{S_q S_o - S_q \sum_{n=1}^{M} O_{pq} S_q}{M (N S_q^2 - S_q^2)}
\]

III. ESTIMATION OF THE SOURCE POSITION

Without losing the generality, the three sensors are positioning at a planar surface at the coordinates: \((0,0), (x_1,0), (x_2,y_2)\). Assuming source position at \((x,y)\), the Euclidean distance between source and sensors are:

\[
r_1(x,y) = \sqrt{x^2 + y^2} \\
r_2(x,y) = \sqrt{(x - x_1)^2 + y^2} \\
r_3(x,y) = \sqrt{(x - x_2)^2 + y - y_1^2}
\]

The accumulated square error between the estimated and the actual source-sensors Euclidean distance is:

\[
E(x,y,G) = \sum_{i=1}^{3} (r_i(x,y) - R_i \cdot G)^2
\]

The local optimum solutions can be derived by the following system of non-linear equations:

\[
\frac{\partial E}{\partial x} = \sum_{i=1}^{3} \frac{\partial E}{\partial r_i} \frac{\partial r_i}{\partial x} - (r_2 - R_2 \cdot G) \sum_{i=1}^{3} \frac{x - x_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2}} = 0
\]

\[
\frac{\partial E}{\partial y} = \sum_{i=1}^{3} \frac{\partial E}{\partial r_i} \frac{\partial r_i}{\partial y} - (r_2 - R_2 \cdot G) \sum_{i=1}^{3} \frac{y - y_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2}} = 0
\]

\[
\frac{\partial E}{\partial G} = \sum_{i=1}^{3} R_i (r_i - R_i \cdot G) = 0 \Rightarrow G = \frac{R_1 r_1 + R_2 r_2 + R_3 r_3}{R_1^2 + R_2^2 + R_3^2}
\]

The solution of (19), (20), and (21) cannot be derived in a close form, but gradient-based iterative methods can be used to derive a local minimum solution.

The convergence efficiency and the computational complexity of the gradient methods depend strongly on the initial assumption. In case where the distance estimation is error-free, the following equations are valid:

\[
r_1^2(x,y) = x^2 + y^2 = R_1^2 G^2
\]

\[
r_2^2(x,y) = (x - x_1)^2 + y^2 = R_2^2 G^2
\]
\begin{equation}
    r_i^2(x, y) = (x - x_i)^2 + (y - y_i)^2 = R_i^2G^2. \tag{24}
\end{equation}

The source position problem can be solved directly by replacing the \(x^2+y^2\) equivalent of (22) into the right part of (23) and (24). The system of equations is simplified to:

\begin{equation}
    R_1^2G^2 - 2xx_1 + x_1^2 = R_2^2G^2, \tag{25}
\end{equation}

\begin{equation}
    R_1^2G^2 - 2xx_2 + x_2^2 - 2yy_2 + y_2^2 = R_3^2G^2. \tag{26}
\end{equation}

Thus, the source position is given by the following Cartesian coordinates:

\begin{equation}
    x = \frac{x_1^2 + R_i^2G^2 - R_2^2G^2}{2x_1}., \tag{27}
\end{equation}

\begin{equation}
    y = \frac{R_1^2G^2 - 2x_2 + x_2^2 + y_2^2 - R_3^2G^2}{2y_2}. \tag{28}
\end{equation}

It is easily verified that (27) and (28) satisfy (22), (23) and (24).

In case where the material constant \(G\) is known and a low error estimation of the source-sensor distances can be obtained, the use of the sensor position given by (27) and (28) ensure initial estimation at the neighbor of the real global minimum. Thus, the gradient-based methods can be accelerated and the convergence to a “bad” local minimum can be avoided.

IV. CONCLUSIONS

Taking into account that AE is among the most important and popular non-destructive testing methods for materials and constructions, the accurate source localization of AE events allowing real-time detection of failure parts. In complex and large scale structures, where multiple sensors are installed, the direct estimation of the source-sensors distance and the AE event-time allow real-time monitoring.